

Profit Maximization through Online Advertising Scheduling for a Wireless Video Broadcast Network

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Abstract—In this paper, we address the problem of how to make the wireless service provider (WSP) earn profits in a wireless video broadcast network with consideration of advertisement insertion. At the beginning, this study examines the profit components by analyzing traffic provision and advertisement insertion. This study considers using two components for profit maximization—one is the function for allocating video rates, and the other is the function for inserting advertisement duration. The maximum achievable profit depends on joint optimization of optimal video-rate vectors and advertisement-duration vectors, which are usually computationally intensive. To resolve such a complexity problem, this work also proposes an effective algorithm for joint optimization. First, the overall profit is formulated as the solution of four local optimization problems through horizontal and vertical decomposition. Second, a theoretic polymatroidal framework is introduced in our work for optimization as this framework is proved effective in profit maximization of multiuser systems. Third, this study shows that the overall profit can be maximized by finding the optimal profit points on the boundary of the rate and duration regions. As a result, the optimum points and the total profit can be obtained through a hierarchical greedy algorithm. Experimental results demonstrate that the proposed method is capable of making maximum profits for WSPs in a wide range of broadcasting rates.

Index Terms—Video, broadcasting, profit, polymatroid, advertisement

1 INTRODUCTION

VIDEO streaming is becoming the dominate traffic in the current and future Internet and wireless networks. The Cisco Visual Networking Index predicted that video traffic accounted for 53 to 69 percent of the global mobile data traffic during 2013 to 2018 [1]. Although wireless server providers (WSPs) offer various quality of services (QoS), most users still download video streams or watch on-line videos for free. Consequently, it is desirable to capitalize the video traffic via advertisements in the future.

With the deregulation of telecommunication industries, future wireless users tend to choose free services to obtain the cost-effective experiences. Unlike conventional commercial broadcasting on television or radio, which occupies the entire available subbandwidth, the focus of wireless broadcasting networks is on the utilization of resources since the

bandwidth is limited and shared. Accordingly, setting prices for traffic provision or advertisements draws much attention from WSPs because such a policy improves their profits. A recent work [2] showed that appropriate scheduling for targeted advertisement insertion is more beneficial and profitable for web-based service providers. However, one particular challenge in wireless systems is how to maximize the profit from advertisements and service provision because excessive advertisements have a negative influence on the number of subscribers. Hence, how to stimulate more user consumptions while advertisements are inserted into the video content is critical to maintain the prior concerns.

1.1 Background

Conventionally, to cope with the varying characteristics of wireless channels in broadcast/multicast services, different methods for increasing robustness and QoS were developed. On the side of video sources, scalable video coding (SVC) [3], layered video coding [4], [5], and adaptive video coding [6], [7], [8] with scalable support were widely employed to provide higher quality, flexibility, and scalability. Regarding the joint source and channel side, since video data are sensitive to transmission failure, streaming should be more reliable. Therefore, joint coding [9], [10], [11] and optimal rate allocation [12], [13] techniques become a feasible solution as these approaches provided various protection levels for video data generating, depending on importance and channel conditions. On the joint modulation side, SVC combined with adaptive modulation and coding schemes (MCS) [14], or combined with cooperative multiple-input-multiple-output (MIMO) [15], [16], [17],

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provided an effective cross-layer solutions for video streaming in heterogeneous devices under wireless broadcast/multicast environments. As for quality-of-experience (QoE) support, recent research on QoE-driven video broadcasting [8], [12], [18], [19] showed better utilization of limited resources for multiuser heterogeneous requirements, with joint consideration of channel characteristics and current available resources. Mobile video broadcasting services [20] are expected to become a popular application for wireless network operators. Unfortunately, when it comes to how to benefit from the wireless video traffic, few systematic studies have been conducted and published. Besides, profit-driven considerations were usually ignored in the related research on conventional wireless video broadcasting. More specifically, researchers did not consider how to capitalize the video traffic via advertisement income.

Recently, Li et al. [14] proved that when SVC was applied to wireless broadcast/multicast streaming with consideration of heterogeneous characteristics, the total utility maximization of a broadcasting system was a nondeterministic polynomial-time hard (NP-hard) problem. The authors indicated that the solution for a profit-driven video broadcasting depended on the following four factors—available resources, the requirement for QoE, the channel conditions of multiple end-users (EUs), and the profit generated based on traffic provision and advertisement insertion. These factors correspondingly deepened the difficulty of realizing a profit-driven wireless video broadcasting system. To overcome such difficulty, this work proposes using polymatroidal structures for solving the profit-driven problem as the original problem is close to the problem of polymatroid intersection. Furthermore, polymatroidal structures have been proved effective [21], [22] as it can be easily generalized when complex constraints are present. Accordingly, polymatroidal structures have been widely used in dynamic resource-allocation problems, especially in networks and multiuser communication systems [23].

1.2 New Challenges

In video broadcasting systems, advertisement insertion is attracting much attention from WSPs because such a policy improves their profits. However, it is still challenging how the QoE of EUs is guaranteed, while diverse requirements are jointly fulfilled. The following new challenges arise due to the presence of profits with advertisement insertion in wireless video broadcasting systems.

In multivideo content broadcasting, since video content in different scalable domains has various rate distortions, a diversity of content rates during broadcasting obviously exist. In other words, if each video content is broadcasted based on a video rate, then when multicontents are broadcast, the system can build a histogram or a vector of video-rate distribution. Allocation of resources according to such video-rate vector apparently affects the profit performance. Typically, more broadcasting contents represent more opportunities for advertisement insertion. Although this appears straightforward to implement, the actual profit model is more complicated than expected. For example, in multivideo content broadcasting, it is difficult for a WSP to determine whether it should insert more advertisements to ensure profits or provide more contents to retain

subscriptions. This is because there is a tradeoff between the number of contents, the number of subscribed users, the quality of video services, and the profit of the WSP.

In a multiuser environment, where videos are broadcasted over wireless channels, approaches like forward error correction (FEC) and unequal error protection (UEP) are usually employed to provide QoS-guaranteed video transmissions as channel conditions change with time. Thus, it takes additional effort for a WSP to impose error protection to provide reliable transmissions, which consequently results in the increase of costs.

In profit-driven video broadcasting, the conflict between the economic efficiency and consumptive experience of users usually exists. This is because, intuitively, EUs always wish to pay less for the services, whereas the WSP expects to earn as many revenues as possible. High prices of traffic provision however decreases the number of EUs. A new profit model is therefore developed—advertisement insertion. The growing popularity of online video services provides a good opportunity for advertisers to reach their customers. This in turn increases the revenues for advertisers and WSPs as the system providers usually charge advertisers based on the number of EUs. However, unlike the interactive advertisement in online video applications, it is difficult to encourage more EUs to watch advertisements voluntarily in wireless broadcasting. Thus, long advertisement-duration inevitably decreases the number of EUs. Therefore, there is a tradeoff between the advertisement duration and the number of EUs.

1.3 New Formulation and Contributions

This paper is around the above three challenges. In the following sections, we present a solution for the problem of how to make the WSP maximize its profits through advertisement insertion with consideration of QoE guarantees. We consider a situation where the WSP sells limited wireless resources to heterogeneous users who have freedom to choose the services, while the WSP tries to make more profits through advertisements. We investigate how EUs pay for video services they purchase under a QoE measure and how WSPs set prices for the traffic and advertisement insertion. We consider the general case where heterogeneous EUs have different experiences and various channel conditions. The EUs have the right to reject or keep receiving video services according to their willingness. Indeed, it is unclear how a wireless video broadcasting has maximum profit under dynamic EUs connection conditions. Thus, we first give explicit definitions to the above three challenges as follows.

First, to tackle the diversity effectively in multivideo content broadcasting, this work proposes an approach consisting of joint profit optimization, source adaptation, and advertisement scheduling for multiuser wireless broadcasting. We formulate the profit maximization problem as a hierarchical optimization problem. By exploiting the content diversity of users, the limited broadcasting resources can be efficiently allocated. Furthermore, the aggregate profits of traffic provision and advertisement insertion can be both maximized while the QoE requirements of heterogeneous users are satisfied at the same time.

Second, for the conflict between the economic efficiency and consumptive experience of users in multiuser

broadcasting systems, in this work, the cost of traffic provision is stated as a constraint. The goal is to find a solution that maximizes collected profits so that traffic costs do not exceed a preset threshold. Furthermore, by analyzing the relation among video source rates, additional error protection rates, and user channel conditions, the cost problem is transformed to the problem of reliable video service provision. A hybrid mechanism between profit maximization and resource allocation is proposed and established to ensure the feasibility of the solution.

Third, for the profit-driven video broadcasting, we carefully choose the adjusting parameters to balance the relation among the profit, the rate, and scheduling regions. This tradeoff becomes more important as the number of video EUs increases.

In this work, profit maximization through advertisement insertion is studied for video traffic-based broadcasting, where limited wireless resources are allocated to heterogeneous users. Furthermore, this work also investigates how EUs select video services under a QoE measure and how WSPs should set prices for the traffic provision and advertisements insertion. Finally, a general case is examined when EUs have different QoE preferences and channel conditions.

This subsection introduces a comprehensive polymatroidal structure and summarizes how the overall profit can be maximized based on the proposed idea. To the best of our knowledge, this is by far the first attempt to combine both resource allocation and advertisement scheduling problems for wireless video broadcasting. The contributions of this work are summarized as follows.

- *Hybrid-domain profit framework.* For a hybrid system with resource-allocation and advertisement scheduling, the existence of optimal profit region depends on two factors. One is the video-rate allocation with consideration of the QoE of heterogeneous EUs, and the other is the scheduling of advertisement insertion with consideration of the number of EUs that are serviced. This study shows that the profit does not always increase the available bandwidth with the longer advertisement insertion, and/or better QoE provision. To prove this, an analytical optimal solution is provided in this work.
- *Profit optimization through hierarchical decomposition.* This study presents a dynamic profit-optimization approach that can simultaneously model—1) the relation between advertisement insertion and subscribers as well as 2) the association between the clients and the cost of WSPs. An objective function is therefore derived and proposed to express those two relations in terms of allocated broadcasting rates and scheduled advertisement duration. Moreover, the constraints prescribed by channel conditions, available bandwidth, advertisement gross, and user QoE are also considered in the proposed objective function. Our results indicate that WSPs can simply classify their EUs into two groups—one prefers better QoE with short-duration and fewer advertisements, and the other favors the lower and inexpensive bandwidth with a little bit more advertisements.

- *Joint optimization through polymatroidal structure.* This study develops a hybrid polymatroidal structure for profit modeling that simultaneously considers the factors of video-rate allocation and advertisement insertion. The profit can be maximized based on the total rates and the total duration in the entire bandwidth by using the polymatroid optimization. Also, the maximum achievable profit can be explicitly obtained in a greedy manner by respectively finding the optimal rate vector (i.e., the rate distribution of the current streaming videos) and advertisement-duration vector (i.e., the durational distribution of the current streaming videos) on the facets formed by the rate and the duration.

The rest of the paper is organized as follows. Section 2 describes the general framework of the profit model. Section 3 then describes how to solve the profit maximization through hierarchical decomposition. In Section 4, the rate allocation and advertisement scheduling are formulated as a hybrid polymatroidal structure. Section 5 shows how to obtain the maximum profit by using polymatroid optimization. Sections 6 and 7 give the simulation results and conclusion.

2 SYSTEM MODEL

Consider a single-cell broadcasting network, where S video contents (i.e., videostreams) are being broadcast to N users. The user population is dynamic, and videos on demand and advertisement insertion are both considered in this circumstance. Each video content s is connected with N_s heterogeneous users, whose channel conditions and screen features are different. Notably, $s \in \{1, \dots, S\}$ and $\sum_{s=1}^S N_s = N$. Besides, reception performance of user n is characterized as an erasure channel rate ϵ_n . To satisfy the requirements of heterogeneous EUs, scalable video coding is employed to provide layered and scalable video support.

Typically, a video service is regarded as a successful transmission if an EU is satisfied with the received quality. The EU pays the WSP for the video services, and the WSP obtains the revenue by providing reliable services for multiple heterogeneous EUs. To ensure QoE, a broadcasting system usually needs to adapt the transmission to heterogeneous connecting environments by considering the video quality, transmission rates, error protection, and traffic provision strategies at the same time. This correspondingly increases the cost of a WSP. What is more, to offer more choices to users, the WSP has to provide flexible video broadcasting for heterogeneous EUs by adopting joint SVC and adaptive layered broadcasting. In such a condition, even if the content that is being requested is the same, the served quality is different from an EU to another, which therefore requires an effective method for WSPs to fully model the cost. The detailed model is depicted later in this work.

As advertisement insertion becomes a trend of business model for WSPs, this work lays emphasis on such a topic. In this study, the behavior of advertisement insertion is modeled as a set of insertion S with finite slots τ_s . The EUs demand the video services by sending a request to the WSP. If the WSP wants to earn revenues by advertisement insertion, the WSP needs to devise an appropriate way so that

clients can accept insertion. In our scenario, before advertisements are inserted into the S contents at the advertisement slots (e.g., at the beginning of each content during playback), the broadcasting system usually slices advertisements into segments and converts them into SVC streams. Therefore, all the properties of the original SVC stream can be maintained in the content¹. Based on such a scenario, this study discusses how to maximize the profit by considering the tradeoff between video traffic provision and advertisement insertion, especially when the limited available bandwidth and video-quality guarantees are required. To derive an objective function for profit maximization, this study analyzes the major characteristics of the overall profit in wireless settings. Then, the profit region and broadcasting model are built based on these characteristics in this paper.

2.1 Revenue and Cost Analysis through Traffic Provision

The EU demands video contents by making a request to a WSP. Let $u_{s,n}^{\text{WSP}(r)}(r_s)$ be the revenue of the WSP when content s is successfully broadcast to EU n at a video rate r_s . Assuming the WSP earns an average revenue per unit rate, we can express the revenue in term of content price a_s (i.e., the price of content subscription) as

$$u_{s,n}^{\text{WSP}(r)}(r_s) = a_s r_s. \quad (1)$$

On the side of EU, the video quality is usually defined as a utility function $u_{s,n}^{\text{EU}}(r_s)$ [24]

$$u_{s,n}^{\text{EU}}(r_s) = \theta_s \ln(1 + r_s), \quad (2)$$

where θ_s represents the willingness to pay of content s .

Theoretically, the system cost derives from the following four parts [25], [26], [27]: i) Deployment costs because the WSP needs to invest in infrastructure before operation, ii) operating expenses due to the service provision, iii) current usage-based or congestion-based degree, etc., and iv) maintenance expenses as the broadcasting system requires management, administration, and long-term maintenance. Besides, the WSP needs to provide the flexible protection for reliable video transmissions in wireless environments. For simplicity, we assume that the cost is dependent on the following factors—the number of users, the rate r_s of video data, and the rate γ_s of error protection data. When the WSP provides a video service for an EU, the cost is given by

$$c_{s,n}^{\text{WSP}(r)}(r_s) = w_s(n_s) \cdot (r_s + \gamma_s), \quad (3)$$

where $w_s(n_s)$ is the cost between the WSP and n_s EUs.

The total profit of a WSP depends on many factors. Since it is difficult to accurately estimate all the costs associated with carrying traffic, this paper focuses on the major factors that are related to the revenue generated from EUs and the above costs of wireless video provision. The total profitability generated from n EUs based on the s^{th} video is then described as

$$\text{Profit}_{s,n}^{\text{WSP}(r)} = u_{s,n}^{\text{WSP}(r)}(r_s) - c_{s,n}^{\text{WSP}(r)}(r_s). \quad (4)$$

1. Unlike the work in [2], this study does not discuss the content of advertisements (i.e., targeted advertisements).

In the wireless scenario, the transmitter employs flexible FEC to provide reliable video transmissions. Most of the existing literature focused on devising effective protection strategies, such as [28] and [11]. Rather than using those protection strategies, this work utilizes Fountain codes to provide flexible error protection in multiuser broadcasting systems. Let ε_{s,n_s} be the channel condition of EU n , who demands the content s . With an appropriate design [29], [30], the reliable video transmission condition in multiuser broadcasting can be roughly estimated by $\gamma_s \geq r_s \left(\frac{1+\delta}{1-\hat{\varepsilon}_{s,n_s}} - 1 \right)$, where δ is a fixed small positive value, $\hat{\varepsilon}_{s,n_s}$ represents the threshold. Such a function indicates that those EUs, whose channel states satisfy $\varepsilon_{s,n_s} \leq \hat{\varepsilon}_{s,n_s}$, reliably receive video streams in a rate r_s . Then, for the wireless video broadcasting system, the WSP can maximize its revenue by allocating the available bandwidth R among multicontent broadcasting, on the premise of reliable traffic provision and a certain quality u_s^0 guarantee. Equation (4) implies that

$$\text{Profit}^{\text{WSP}(r)} : \sum_{s=1}^S \sum_{n=1}^{n_s} a_s r_s - w_s(n_s) \cdot (r_s + \gamma_s), \quad (5)$$

$$\text{s.t. } u_{s,n}^{\text{EU}}(r_s) \geq u_s^0, \quad (6)$$

$$\sum_{s=1}^S r_s + \gamma_s \leq R_{\text{Budget}}, \quad (7)$$

$$r_s \leq R_s, \quad (8)$$

$$n_s \leq N_s, \quad (9)$$

$$\gamma_s \geq r_s \left(\frac{1+\delta}{1-\hat{\varepsilon}_{s,n_s}} - 1 \right), \quad (10)$$

$$r_s, \gamma_s, \hat{\varepsilon}_{s,n_s}, n_s \geq 0, \quad (11)$$

where (6) indicates the quality guarantee level u_s^0 ; (7) gives the available bandwidth budget R_{Budget} ; (8) shows that the source-rate adaptation supported by the broadcasting; (9) demonstrates the basic condition in general multiuser communications, in which the actual number of EUs being serviced might be fewer than the original number of EUs requesting services; (10) specifies the reliable multiuser broadcasting condition.

2.2 Profit through Advertisement Insertion

Nowadays, advertising has become the major income for most service providers as the price of traffic gradually decreases. The WSPs place advertisements in their channels and broadcast them to their EUs. Then, the WSPs charge their advertisement clients based on the broadcasting frequency, the duration, and the schedule (i.e., prime time or fringe time). For example, the price of an advertisement inserted in a popular event or in a prime time is higher than that inserted in a usual event due to more audiences and higher popularity. Notably, an advertisement service usually requires the consent of EUs. Accordingly, long-duration or high-frequent insertion results in the decreasing number of EUs. Based on such a concept, the revenue $u_{s,n}^{\text{WSP(Ad)}}(\tau_s)$ gained from advertisement services can be characterized as

a function according to the advertisement duration τ and the content price b_s (i.e., popular events or not):

$$u_{s,n}^{\text{WSP(Ad)}}(\tau_s) = b_s \tau_s. \quad (12)$$

The risk of advertisement services mainly comes from the tolerance of the EUs because comparing with the traffic provision or broadcasting, the cost of advertisement insertion can be ignored. Therefore,

$$\text{Profit}^{\text{WSP(Ad)}} : \sum_{s=1}^S b_s \tau_s, \quad (13)$$

$$\text{s.t.} \quad \sum_{s=1}^S \tau_s \leq T_{\text{budget}}, \quad (14)$$

$$\tau_s \leq T_0, \quad (15)$$

where (14) shows that the amount of advertisement duration is limited to T_{budget} . The condition (15) demonstrates that the total duration for each content should not exceed the utilization threshold T_0 .

2.3 Properties

(1) *Relation between n_s and τ_s*

Observation 1. Longer duration of advertisements τ_s decreases the number of EUs n_s .

It is straightforward to see that the long-duration advertisement leads to more content switching. The willingness of EUs to wait for videostreaming and to keep watching advertisements apparently has a tolerance degree. According to the work by Ha et al. [31], the number of EUs, who stay tuned while watching commercials in a time-frame τ , can be calculated as follows:

$$n_s(\tau_s) = \begin{cases} N_s, & \tau_s \leq \tau_s^{\text{th}}, \\ \frac{1}{\lambda_\rho \left(\frac{\tau_s - \tau_s^{\text{th}}}{\tau_s^{\text{th}} + 1}\right)^\rho} N_s, & \tau_s > \tau_s^{\text{th}}, \end{cases} \quad (16)$$

where ρ is a parameter that measures user patience, λ_ρ denotes an appropriate normalization constant, and N_s represents the total number of EUs, who demand the content s .

(2) *Relation between w_s and n_s*

Observation 2. The cost w_s increases drastically when the n_s is beyond a threshold N_s^{th} .

One example is showed in Fig. 2. The transmission capacity and corresponding available bandwidth are usually limited. When mass EUs demand a video service at the same time, a large number of EUs could stay in an extremely worse channel condition. If the transmission becomes unreliable, EUs might lose their patience and quit. In this case, it is assumed that the broadcasting system intends to keep as many users as possible by broadcasting its services at a high cost. Thus, the service capacity of a wireless broadcasting system has been limited to the saturated number of EUs. The saturation condition is modeled as follows:

$$w_s(n_s) = \begin{cases} w_0, & n_s \leq N_s^{\text{th}}, \\ w_0 e^{\frac{n_s}{N_s^{\text{th}}}}, & n_s > N_s^{\text{th}}, \end{cases} \quad (17)$$

where w_0 is an exponential parameter that measures the cost of a normal case, and N_s^{th} presents the number of supporting EUs when the broadcasting system is initially operated.

2.4 Hybrid Model for Profit Maximization Formulation

The total profit $\text{Profit}^{\text{WSP}}$ of a broadcasting system can be built as a hybrid model that considers both traffic provision profits $\text{Profit}^{\text{WSP(r)}}$ and advertisement insertion profits $\text{Profit}^{\text{WSP(Ad)}}$, based on the aforementioned formulations:

$$\text{Profit}^{\text{WSP}} : \max \sum_{s=1}^S \left(b_s \tau_s + \sum_{n=1}^{n_s} a_s r_s - w_s(n_s) \cdot (r_s + \gamma_s) \right), \quad (18)$$

$$\text{s.t.} \quad u_{s,n}^{\text{EU}}(r_s) \geq u_s^0, \quad (19)$$

$$\sum_{s=1}^S r_s + \gamma_s \leq R_{\text{Budget}}, \quad (20)$$

$$r_s \leq R_s, \quad (21)$$

$$n_s \leq N_s, \quad (22)$$

$$\gamma_s \geq r_s \left(\frac{1 + \delta}{1 - \hat{\epsilon}_{s,n_s}} - 1 \right), \quad (23)$$

$$\sum_{s=1}^S \tau_s \leq T_{\text{budget}}, \quad (24)$$

$$\tau_s \leq T_0, \quad (25)$$

$$r_s, \gamma_s, \hat{\epsilon}_{s,n_s}, n_s \geq 0, \quad (26)$$

$$\text{var.} \quad \mathbf{r}, \boldsymbol{\tau}. \quad (27)$$

3 PROFIT MODEL AND DECOMPOSITION

3.1 Hierarchical Decomposition

This section introduces the decomposition method proposed by Palomar and Chiang [32] for analyzing the objective function in Eq. (18). This work adopts the idea of Palomar and Chiang with several modifications to support hierarchical decomposition of the component structure among profits. According to the theory by Palomar and Chiang, the component structure can be divided into two subproblems.

Let $\Theta(\mathbf{r}, \boldsymbol{\tau}) = \max_{\mathbf{r}, \boldsymbol{\tau}} [\text{Profit}^{\text{WSP}}]$ be the primal objective function, where $\boldsymbol{\tau} = [\tau_1, \dots, \tau_S]$ and $\mathbf{r} = [r_1, \dots, r_S]$. Therefore,

$$\Theta(\mathbf{r}, \boldsymbol{\tau}) = \Theta_{(\text{Ad})}(\boldsymbol{\tau}) + \Theta_{(\text{r})}(\mathbf{r}), \quad (28)$$

where $\Theta_{(\text{Ad})}(\boldsymbol{\tau}) = \text{Profit}^{\text{WSP(Ad)}}$ and $\Theta_{(\text{r})}(\mathbf{r}) = \text{Profit}^{\text{WSP(r)}}$. Obviously, $\Theta_{(\text{Ad})}(\boldsymbol{\tau})$ and $\Theta_{(\text{r})}(\mathbf{r})$ are the individual results of Eqs. (13) and (5), respectively. The decomposed

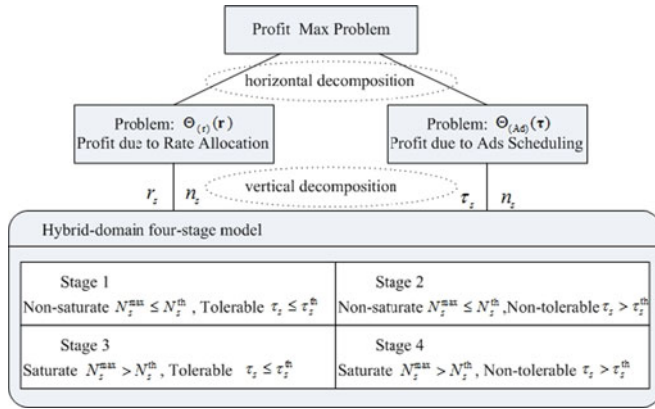


Fig. 1. Decomposition of profit maximization problem.

subproblems $\Theta_{(Ad)}(\tau)$ and $\Theta_{(r)}(r)$ respectively control the variable vectors, i.e., τ and r . According to the reliable broadcasting condition in Eq. (10), the broadcasting rate r_s received by n_s users is differentiated by the threshold $\hat{\epsilon}_{s,n_s}$, which is determined by n_s . Based on observations 1 and 2, n_s can be formulated as a function of τ_s . In the meantime, n_s and τ_s affect the cost and the resulting profit $\text{Profit}^{WSP(r)}$. Such constraints introduces the coupling effect to the profit maximization problem, so that the original problem can be solved by using decomposition methods. The reason why this paper applies hierarchical decomposition is its efficiency and effectiveness. The illustration of the hierarchical decomposition is shown in Fig. 1.

3.2 Hybrid-Domain Hierarchical Model

Assume that there are two basic thresholds in each broadcasting video content—One is user saturation N_s^{th} , and the other is user tolerance τ_s^{th} . To simplify the exposition, the following content discusses the profit problem by introducing the saturation domain and the tolerance domain firstly. The basic profit maximization problem can be classified into four disjoint domains:

- (1) Nonsaturated $N_s \leq N_s^{th}$ and tolerable $\tau_s \leq \tau_s^{th}$. In this condition, the number of total EUs requesting services is not beyond the capacity of a broadcasting system. At the same time, the EUs accept and tolerate a small piece of advertisement insertion. This is the normal and ideal operational scenario for a broadcasting system (see Section 5.1).
- (2) Nonsaturated $N_s \leq N_s^{th}$ and nontolerable $\tau_s > \tau_s^{th}$. The duration of advertisement insertion is beyond the tolerance degree of users. One scenario that frequently comes up is, the WSP inserts long-duration commercials such that users switch the channel or quit the service. However, in such a case, there is still an optimal solution for profit maximization when suitable n_s and τ_s are determined (see Section 5.2).
- (3) Saturated $N_s > N_s^{th}$ and tolerable $\tau_s \leq \tau_s^{th}$. The number of total EUs requesting services is beyond the serving capacity of the broadcasting system. The broadcasting system becomes overloaded such that the cost increases rapidly when the $N_s - N_s^{th}$ users

are serviced. This case usually occurs in popular events, such as the world cup and top selling movies. Like (2), an optimal solution also exists in this case (see Section 5.3).

- (4) Saturated $N_s > N_s^{th}$, nontolerable $\tau_s > \tau_s^{th}$. A large number of EUs are requesting the video services, but users are not willing to tolerate advertisements during broadcasting. When such a circumstance occurs, the system uses a mechanism to lower the number of users requesting services by extending the duration of advertisements and increasing insertions (see Section 5.4).

The WSP maximizes its profit by designing an effective adjusting mechanism to manage the demands from EUs. Since the WSP always has a limited bandwidth resource, it must guarantee that the total demands from EUs are not larger than its serving capacity. The most important, the WSP can obtain the largest benefit with such a mechanism. The following sections present the solution domain by domain by exploiting the profit properties of the polymatroidal structure.

4 RATE ALLOCATION AND ADVERTISEMENT INSERTION SCHEDULING THROUGH HYBRID POLYMATROID PROFIT STRUCTURE

Observations in [22], [23] showed that for a wide class of multiuser systems, particularly in wireless ones, the entire or a subset of the utility region formed an immanent polymatroidal structure. Exploiting the polymatroidal structure could simplify the algorithmic treatment of broadcast/multicast rate regions. In this work, we observe that profit regions exhibit a polymatroidal structure, and the following content then uses this explicit characteristic of the profit region to create a hybrid structure, which can actually solve practical problem in the real application.

4.1 Polymatroid Profit Structure Analysis

Consider a broadcasting scenario, where a WSP remains profitable through adjusting the allocated rates and inserting advertisements. The profit regions are respectively characterized through optimal rate allocation and scheduling policies. Let us denote \mathcal{R}_r as a rate-allocation policy, which is a mapping from the rate space to \mathbb{R}_+^S . Besides, \mathcal{R}_r can be interpreted as the video rates allocated to all S contents. Given the characteristics of EUs, the profit region $\mathcal{P}^{WSP(r)}(\mathcal{R}_r)$ under policy \mathcal{R}_r shows a polymatroidal structure for our broadcasting system. The following theorem substantiates such an interpretation.

Theorem 1. *Given a policy \mathcal{R}_r , the achievable profit region of the system $\mathcal{P}^{WSP(r)}(\mathcal{R}_r)$ is a polymatroid, where*

$$\mathcal{P}^{WSP(r)}(\mathcal{R}_r) = \left\{ \mathbf{P}^{WSP(r)} \in \mathbb{R}_+^S \mid \mathbf{P}^{WSP(r)}(A) \leq \sum_{s=1}^{|A|} \sum_{n=1}^{n_s} a_s r_s - w_s(n_s) \cdot (r_s + \gamma_s), A \subseteq S \right\}. \quad (29)$$

Proof. See Appendix A. □

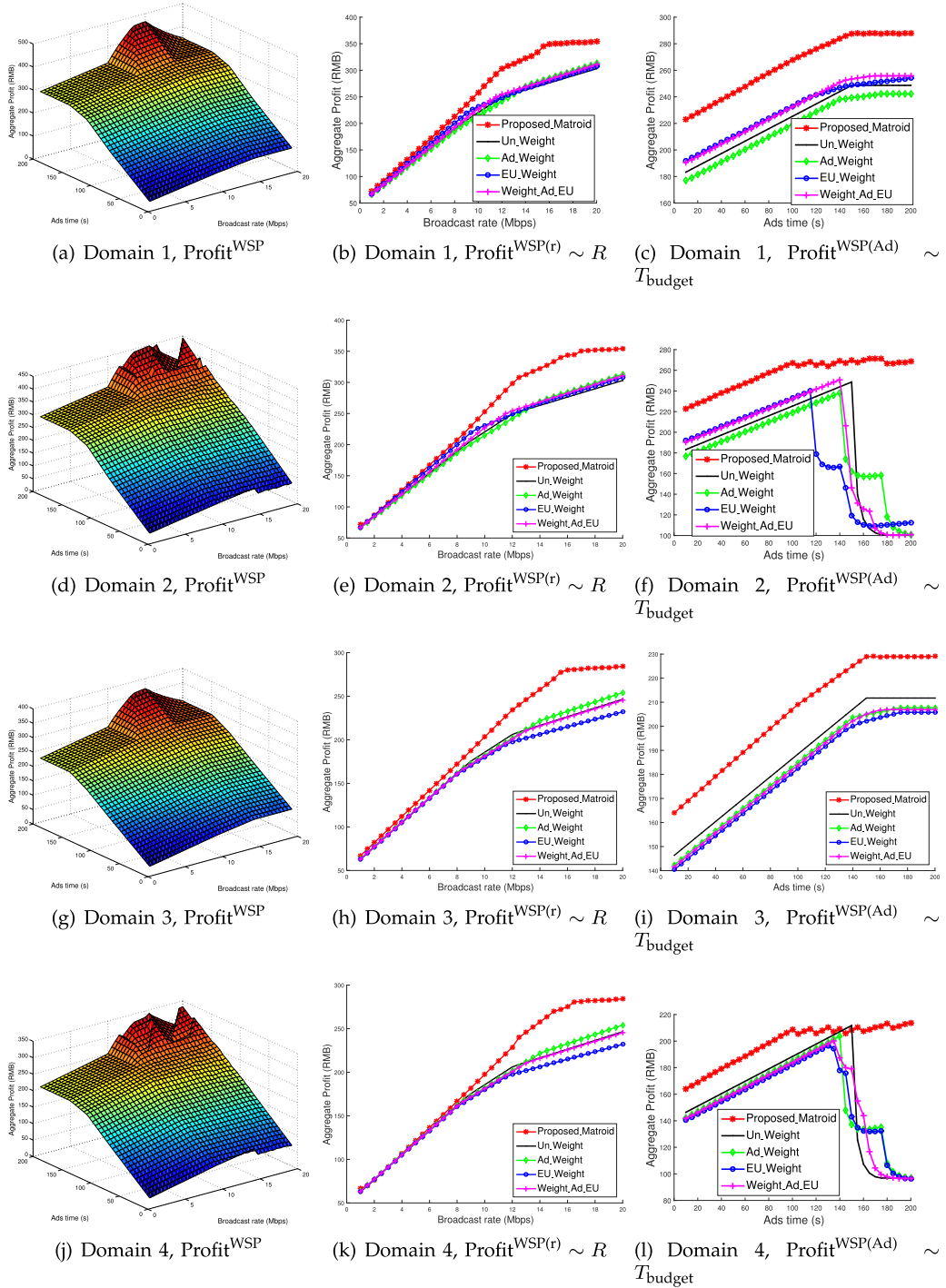


Fig. 2. Performance comparison between the baselines and the proposed strategy for wireless video broadcasting.

Accordingly, let \mathcal{R}_τ represent a duration-allocation policy, which means the total duration of advertisements scheduled to be inserted into all S contents. The subsequent theorem shows that the resulting profit region $\mathcal{P}^{\text{WSP(a)}}(\mathcal{R}_\tau)$ is also a polymatroidal structure.

Theorem 2. Under a scheduling policy \mathcal{R}_τ , the achievable profit region of the WSP $\mathcal{P}^{\text{WSP(a)}}(\mathcal{R}_\tau)$ is a polymatroid, where $\mathcal{P}^{\text{WSP(a)}}(\mathcal{R}_\tau)$

$$= \left\{ \mathbf{P}^{\text{WSP(a)}} \in \mathbb{R}_+^S \mid \mathbf{P}^{\text{WSP(a)}}(B) \leq \sum_{s=1}^{|B|} b_s \tau_s, B \subseteq S \right\}. \quad (30)$$

Proof. The proof is similar to that of Theorem 1 in Appendix A. \square

Based on Theorems 1 and 2, for the problem of Eq. (18), the total profit region for the WSP can be characterized by the following theorem.

Theorem 3. Given policy $(\mathcal{R}_\tau, \mathcal{R}_r)$, the achievable hybrid profit region of the system $\mathcal{P}^{\text{WSP}}(\mathcal{R}_\tau, \mathcal{R}_r)$ is a polymatroid, where

$$\mathcal{P}^{\text{WSP}}(\mathcal{R}_\tau, \mathcal{R}_r) = \mathcal{P}^{\text{WSP(a)}}(\mathcal{R}_\tau) \times \mathcal{P}^{\text{WSP(r)}}(\mathcal{R}_r). \quad (31)$$

Proof. By direct verification. \square

4.2 Profit Region via Hybrid Polymatroidal Profit Structure

This subsection introduces that for a multicontent video broadcasting system, the profit regions can be represented by a broadcasting functions. Notably, the objective of this subsection is to model \mathcal{P}^{WSP} . As the polymatroidal structure of the broadcast functions also induces the polymatroidal structure for the union profit region, the total profit region \mathcal{P}^{WSP} is the union of all the components $\mathcal{P}^{\text{WSP}(r)}$ and $\mathcal{P}^{\text{WSP}(Ad)}$. Thus, it is sufficient to first consider the two components in terms of polymatroidal structure with the broadcasting function. Such two components are respectively stated in the following theorems.

Theorem 4. : *When rates are dynamically allocated according to policy $\mathcal{R} \in \mathcal{F}$, where \mathcal{F} is the set of all feasible rate-control policies that satisfy the rate constraint $\mathcal{F} \equiv \{\mathcal{R} : (\mathbf{R}(A) + \Upsilon(A)) \leq R_{\text{budget}}, \forall s\}$, the profit region with respect to traffic provision can be defined as*

$$\text{WSP}(r) : \mathcal{P}^{\text{WSP}(r)}(\bar{\mathbf{r}}) = \bigcup_{\mathcal{R} \in \mathcal{F}} \mathcal{P}^{\text{WSP}(r)}(\mathcal{R}). \quad (32)$$

Proof. See Appendix B. \square

Theorem 5. *When the duration of advertisements is dynamically allocated based on policy $\mathcal{T} \in \mathcal{G}$, where \mathcal{G} is the set of all feasible duration-control policies that satisfy the duration constraint $\mathcal{G} \equiv \{\mathcal{T} : (\mathbf{T}(P) \leq T_{\text{budget}}, \forall s)\}$, the profit region with respect to advertisement insertion is given by*

$$\text{WSP}(Ad) : \mathcal{P}^{\text{WSP}(Ad)}(\bar{\mathbf{r}}) = \bigcup_{\mathcal{T} \in \mathcal{G}} \mathcal{P}^{\text{WSP}(Ad)}(\mathcal{T}). \quad (33)$$

Proof. The proof is similar to that of Theorem 4 in Appendix B. \square

The following result shows that the total region (i.e., hybrid region) can be written as a union of the profit regions with respect to traffic provision and advertisement insertion.

Theorem 6. *The total profit region of the WSP is given by*

$$\text{WSP} : \mathcal{P}^{\text{WSP}}(\bar{\mathbf{r}}, \bar{\boldsymbol{\tau}}) = \mathcal{P}^{\text{WSP}(r)}(\bar{\mathbf{r}}) \times \mathcal{P}^{\text{WSP}(Ad)}(\bar{\boldsymbol{\tau}}). \quad (34)$$

Proof. By direct verification. \square

4.3 Profit Maximization through Upper Boundary Surface Achieving

Theorems 1, 2, and 3 prove that the total profit region $\mathcal{P}^{\text{WSP}}(\mathcal{R}_\tau, \mathcal{R}_r)$ and its two components $\mathcal{P}^{\text{WSP}(r)}(\mathcal{R}_r)$ and $\mathcal{P}^{\text{WSP}(a)}(\mathcal{R}_\tau)$ exhibit polymatroidal structures. Theorems 4, 5, and 6 give a hybrid profit region under the rate-allocation and advertisement-scheduling policies. As the structure of profit regions is validated, in the following content, this study makes use of the polymatroidal structure to characterize the boundary of the profit region. Although a similar version of the following results was obtained in [23], the difference between that paper and

this work is that the region \mathcal{P}^{WSP} of this study is controlled by both $\boldsymbol{\tau}$ and \mathbf{r} .

Definition 7. *The boundary surface of $\mathcal{P}^{\text{WSP}}(\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}})$ is the set of those rate-duration pairs $(\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}})$ such that no component can be increased while the other components remain fixed and in $\mathcal{P}^{\text{WSP}}(\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}})$.*

Let π be a permutation of the set $E = \{1, \dots, S\}$, which represents the element of the set E located in the i^{th} position after the permutation [33]. The solution to the optimization problem can be found by locating the extreme point of polymatroids and using permutation [34], [23]. Based on such a concept, for the two strategies, rate allocation and duration scheduling, we have

4.3.1 Permutation Based on Rates

$\Pi_r = \{\pi_r(1), \dots, \pi_r(S)\}$, where $\pi_r(s) := (a_{\pi_r(s)} - \frac{w_{\pi_r(s)}}{1 - \varepsilon_{\pi_r(s)}})N_s$ and $(a_{\pi_r(1)} - \frac{w_{\pi_r(1)}}{1 - \varepsilon_{\pi_r(1)}})N_{\pi(1)} > \dots > (a_{\pi_r(S)} - \frac{w_{\pi_r(S)}}{1 - \varepsilon_{\pi_r(S)}})N_{\pi(S)}$.

4.3.2 Permutation Based on Advertisements

$\Pi_{Ad} = [\pi_{Ad}(1), \dots, \pi_{Ad}(S)]$, where $\pi_{Ad}(s) := b_{\pi(s)}$ and $b_{\pi(1)} > \dots > b_{\pi(S)}$.

Thus, the maximum of the profit lies at the points of the boundary surface because any other point in the profit region should not fall outside the boundary surface. Then, the corresponding optimal rate-control and duration-scheduling policies can be reduced to a boundary-achieving problem expressed in the following theorem.

Theorem 8. *The boundary surface of $\mathcal{P}^{\text{WSP}}(\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}})$ is the closure of all pairs such that $(\boldsymbol{\tau}^*, \mathbf{r}^*)$ is a solution to the optimization problem*

$$\max_{\boldsymbol{\tau}, \mathbf{r}} \mathbf{P}^{\text{WSP}} - \lambda_1 \boldsymbol{\tau} - \lambda_2 \mathbf{r} \quad \text{s.t.} \quad \mathbf{P}^{\text{WSP}} \in \mathcal{P}^{\text{WSP}}(\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}}). \quad (35)$$

Proof. See Appendix C. \square

Consequently, there exist rate vectors and duration vectors in the polymatroid-based broadcasting network, which satisfies Eq. (18). That is, the maximal broadcast profit can be found on the boundary surface. The rate vectors and duration vectors are from the sum-profit facet of the polymatroid, and such facet can be found by locating the extreme points.

5 POLYMATROID PROFIT STRUCTURE AND PROFIT MAXIMIZATION

As the transmission capacity is usually limited, there is a maximum number of supporting EUs for a general broadcasting system. The system profit is an accumulative profit generated from $\sum_s n_s$ EUs, who demand diverse video contents and are already serviced. Therefore, such a condition $n_s \leq N_s$ holds. Recall that the advertisement insertion affects n_s for all the S video services.

Consequently, two interactions exist between the WSP and EUs, which result in n_s and the corresponding aggregate profit. In Section 3.2, we characterize the interactions as

a hybrid-domain hierarchical model. Since the WSP is capable of controlling the access of each EU, it is possible for the WSP to manage its profit by charging the EUs that are already serviced.

With the polymatroid profit structure, the optimal solution can be found by adding more rates and more advertisement duration until the marginal profit becomes negative. Once the optimal solution is obtained, the WSP can determine which settings should be selected by observing these two variables, τ_s and r_s , with the highest marginal profit.

The next discussion analyzes the system profit in four domains, respectively.

5.1 Domain 1: Nonsaturated $N_s \leq N_s^{\text{th}}$ and Tolerable $\tau_s \leq \tau_s^{\text{th}}$

The first domain represents that the broadcasting system still has available and sufficient resources so that the entire system is not overloaded. Therefore, the performance of video streaming is satisfactory, and EUs still have patience to wait for the service.

The objective function in Eq. (18) becomes

$$\begin{aligned} \text{Profit}^{\text{WSP}} &= \sum_{s=1}^S \left(b_s \tau_s + \sum_{n=1}^{N_s} a_s r_s - w_s(n_s)(r_s + \gamma_s) \right) \\ &= \sum_{s=1}^S \left(b_s \tau_s + N_s (a_s r_s - w_s(n_s)(r_s + \gamma_s)) \right). \end{aligned} \quad (36)$$

Equation (36) can be further decomposed into the following independent bi-objective optimization problems:

$$\left\{ \max \sum_{s=1}^S b_s \tau_s, \max N_s (a_s r_s - w_s(n_s)(r_s + \gamma_s)) \right\}. \quad (37)$$

Equation (37) can be directly decomposed into separate two objectives—(i) $\max \sum_{s=1}^S b_s \tau_s$ and (ii) $\max \sum_{s=1}^S N_s (a_s r_s - w_s(n_s)(r_s + \gamma_s))$. Then, the original profit problem in Eq. (18) is rewritten as subobjective optimization (i)

$$\max \Theta_{(\text{Ad})}(\boldsymbol{\tau}), \quad (38)$$

$$\text{s.t. } \boldsymbol{\tau} \cdot \mathbf{1} \leq T_{\text{budget}}, \quad (39)$$

$$\boldsymbol{\tau} \succeq \mathbf{0}, \quad (40)$$

and subobjective optimization (ii)

$$\max \Theta_{(\text{r})}(\mathbf{r}), \quad (41)$$

$$\text{s.t. } (\mathbf{r} + \boldsymbol{\gamma}) \cdot \mathbf{1} \leq R_{\text{budget}}, \quad (42)$$

$$r_s \left(\frac{1 + \delta}{1 - \hat{\varepsilon}_{s, N_s}} - 1 \right) \leq \gamma, \quad \forall s \in \{1, \dots, S\}, \quad (43)$$

$$\mathbf{r}, \boldsymbol{\gamma} \succeq \mathbf{0}. \quad (44)$$

Thus, the profit can be maximized by directly solving the optimal results in (i) and (ii) if the condition matches the first domain. The following theorem substantiates such an interpretation.

Theorem 9. [Optimal Solution in the First Domain] The optimal solution $(\boldsymbol{\tau}^*, \mathbf{r}^*)$ for $\Theta(\boldsymbol{\tau}, \mathbf{r})$ in the first domain is the global optimal solution to the entire problem. Therefore, such a solution is also the optimal solution (i.e., $\boldsymbol{\tau}^*$ and \mathbf{r}^*) to separate subproblems $\Theta_{(\text{Ad})}(\boldsymbol{\tau})$ and $\Theta_{(\text{r})}(\mathbf{r})$, where

$$\Theta(\boldsymbol{\tau}, \mathbf{r}) = \Theta_{(\text{Ad})}(\boldsymbol{\tau}) + \Theta_{(\text{r})}(\mathbf{r}). \quad (45)$$

Proof. See Appendix D. \square

Define the marginal profit functions of $\boldsymbol{\tau}$ and \mathbf{r} as $m_{(\text{Ad})}(\tau_s) = \frac{\partial \Theta(\boldsymbol{\tau}, \mathbf{r})}{\partial \tau_s}$ and $m_{(\text{r})}(r_s) = \frac{\partial \Theta(\boldsymbol{\tau}, \mathbf{r})}{\partial r_s}$, respectively. Based on Eq. (37) and Theorem 9, the marginal profits in the first domain are calculated as

$$m_{(\text{Ad})}(\tau_s) = b_s, \quad (46)$$

$$m_{(\text{r})}(r_s) = \left(a_s - \frac{w_0}{1 - \varepsilon_s} \right) N_s. \quad (47)$$

Example 1. Consider a typical multicontent broadcasting system in the normal operation mode (i.e., the first domain). Assuming the S contents satisfy the condition “ $\forall s \in S$,” namely, $N_s \leq N_s^{\text{th}}$. Also assuming that for each broadcasting content, the advertisement insertion satisfies $\tau_s \leq \tau_s^{\text{th}}$. The maximum profit of broadcasting can be achieved by the optimal permutation, which is shown in Algorithm 1.

Algorithm 1. Optimal Permutation in the First Domain

- 1: **Optimal permutation Π_{Ad}^***
 $\Pi_{\text{Ad}}^* = [\pi_{\text{Ad}}(1), \dots, \pi_{\text{Ad}}(S)]$,
 where $m_{(\text{Ad})}(\tau_{\pi_{\text{Ad}}(s)}) = b_{\pi_{\text{Ad}}(s)}$ and $b_{\pi_{\text{Ad}}(1)} > \dots > b_{\pi_{\text{Ad}}(S)}$
- 2: **Optimal permutation Π_{r}^***
 $\Pi_{\text{r}}^* = \{\pi_{\text{r}}(1), \dots, \pi_{\text{r}}(S)\}$,
 where $m_{(\text{r})}(r_{\pi_{\text{r}}(s)}) = \left(a_{\pi_{\text{r}}(s)} - \frac{w_{\pi_{\text{r}}(s)}}{1 - \varepsilon_{\pi_{\text{r}}(s)}} \right) N_{\pi_{\text{r}}(s)}$ and
 $\left(a_{\pi_{\text{r}}(1)} - \frac{w_{\pi_{\text{r}}(1)}}{1 - \varepsilon_{\pi_{\text{r}}(1)}} \right) N_{\pi_{\text{r}}(1)} > \dots > \left(a_{\pi_{\text{r}}(S)} - \frac{w_{\pi_{\text{r}}(S)}}{1 - \varepsilon_{\pi_{\text{r}}(S)}} \right) N_{\pi_{\text{r}}(S)}$
- 3: **Optimal permutation $(\Pi_{\text{Ad}}^*, \Pi_{\text{r}}^*)$**

5.2 Domain 2: Nonsaturated $N_s \leq N_s^{\text{th}}$ and Nontolerable $\tau_s > \tau_s^{\text{th}}$

Since τ_s can be rewritten as $\tau_s = \tau_s^{\text{th}} + \Delta\tau_s^*$, the increase of $\Delta\tau_s$ results in the decrease of Δn_s and $\text{Profit}^{\text{WSP}(\text{r})}$. Further, $\text{Profit}^{\text{WSP}(\text{Ad})}$ increases as $\Delta\tau_s$ rises. Thus, we can use the incremental scheme to analyze the profit. Note that the original primal problem is decomposed into the components of fixed profits and dynamic profits, where the former is the result of the τ_s^{th} advertisement insertion whereas the latter comes from the fluctuation profit due to $\Delta\tau_s^*$. Consequently, the profit maximization problem is converted to how to utilize the profit fluctuation. The following lemma shows that the primal profit problem is equivalent to the problem of fixed and dynamic profits.

Lemma 10. When $N_s^{\text{max}} \leq N_s^{\text{th}}$ and $\tau_s > \tau_s^{\text{th}}$, the solution for $\text{Profit}_s^{\text{WSP}}(\tau_s, N_s)$ is equivalent to the solution for $\text{Profit}_s^{\text{WSP}}(\tau_s^{\text{th}}, N_s) + \text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s)$, where

$$\text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s) := \text{Profit}_s^{\text{WSP(Ad)}}(\Delta\tau_s, N_s) - \text{Profit}_s^{\text{WSP(r)}}(\Delta\tau_s, \Delta n_s). \quad (48)$$

Proof. See Appendix E. \square

Consequently, the profit maximization has a simple solution, and it can be solved by using the dynamic profit optimization. The following lemma shows the detail.

Lemma 11. *The optimal solution for $\text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s)$ is the same as the optimal solution to $\text{Profit}_s^{\text{WSP}}(\tau_s, N_s)$.*

Proof. By direct verification. \square

The aforementioned two lemmas imply an expending-window strategy, where the WSP can adjust the length of $\Delta\tau_s^*$ to achieve the maximum profit. Algorithm 2 summarizes the expending-window strategy for computing the optimal $\Delta\tau_s$ and Δn_s .

Algorithm 2. Expanding-Window Strategy in the Second domain

1: Characterize the decrement of Δn_s via Eq. (16)

$$\begin{aligned} \Delta n_s &= n'_s(\tau_s)\Delta\tau_s \\ \text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s) &= b_s\Delta\tau_s - \left(1 - \frac{1}{\lambda_\rho(\Delta\tau_s+1)^\rho}\right) N_s r_s \left(a_s - w_s \frac{1}{1-\varepsilon_s}\right) \end{aligned}$$

2: Maximize $\text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s)$ through Lagrangian multipliers

$$\begin{aligned} \Delta\tau_s^* &= \sqrt[\rho+1]{\frac{N_s \cdot r_s \cdot (a_s - w_s \frac{1}{1-\varepsilon_s}) \cdot \rho}{\lambda_\rho \cdot b_s}} - 1 \\ \Delta n_s^* &= n'_s(\tau_s)|_{\Delta\tau_s^*} \Delta\tau_s^* \end{aligned}$$

3: Obtain the optimal profit through the polymatroidal structure $\text{Profit}^{\text{WSP}}$

$$\begin{aligned} &= \sum_{s=1}^S \left(b_s \tau_s^{\text{th}} + b_s \Delta\tau_s^* \right. \\ &\quad \left. + \sum_{n=1}^{N_s - \Delta n_s^*} (a_s r_s - w_s (r_s + \gamma_s)) \right) \end{aligned}$$

Notably, during each increment $\Delta\tau_s$, the component Δn_s should always be increased since it leads to the largest profit of the objective function. Thus, the expending-window strategy is optimal.

Theorem 12. [Optimal Solution in the Second Domain] *The optimal solution (τ^*, \mathbf{r}^*) for $\Theta(\tau, \mathbf{r})$ in the second domain is also the optimal solution \mathbf{r}^* for $\Theta^\tau(\mathbf{r})$ based on Algorithm 1, where*

$$\Theta^\tau(\mathbf{r}) = \Theta(\tau^{\text{th}}, \mathbf{r}) + \Theta(\Delta\tau, \mathbf{r}). \quad (49)$$

Proof. By direct verification. \square

Consequently, the marginal profits in the second domain are

$$m_{(\text{Ad})}(\tau_s) = b_s. \quad (50)$$

$$m_{(\text{r})}(r_s) = \sqrt{\frac{n_s(a_s - \frac{w_0}{1-\varepsilon_s})}{b_s r_s} + b_s n_s(a_s - \frac{w_0}{1-\varepsilon_s})}. \quad (51)$$

Example 2. Consider a multicontent broadcasting system, where the duration of an inserted advertisement is too long such that EUs begin to quit the service. The maximum

profit of the broadcasting system in this case is obtained through the following optimal permutation algorithm.

Algorithm 3. Optimal Permutation in the Second Domain

1: **Optimal permutation Π_r^***

$$\Pi_r^* = [\arg_{\pi(1)} \text{Profit}_{\pi(1)}^{\text{WSP}}, \dots, \arg_{\pi(S)} \text{Profit}_{\pi(S)}^{\text{WSP}}],$$

where $\text{Profit}_{\pi(s)}^{\text{WSP}} = b_{\pi(s)} \tau_s^{\text{th}} + b_s \Delta\tau_s^*$

$$+ \sum_{n=1}^{N_{\pi(s)} - \Delta n_{\pi(s)}^*} r_{\pi(s)} (a_{\pi(s)} - \frac{w_{\pi(s)}}{1-\varepsilon_{\pi(s)}}),$$

$$\forall s \in [1, S]$$

2: **Optimal permutation Π_{Ad}^***

$$\Pi_{\text{Ad}}^* = \{\pi_{\text{Ad}}(1), \dots, \pi_{\text{Ad}}(S)\}, \text{ where } \pi_{\text{Ad}}(s) := \tau_s + \Delta\tau_s^*$$

3: **Optimal permutation**

$$(\Pi_{\text{Ad}}^*, \Pi_r^*)$$

5.3 Domain 3: Saturated $N_s > N_s^{\text{th}}$ and Tolerable

$$\tau_s \leq \tau_s^{\text{th}}$$

Since contents of a popular event (e.g., FIFA world cup) usually result in overloading situations, the cost

$w_s(n_s) = w_0 e^{\frac{N_s}{N_s^{\text{th}}}}$ increases for a practical video broadcasting system. In this case, Eq. (18) is rewritten as

$$\text{Profit}^{\text{WSP}} = \sum_{s=1}^S \left(b_s \tau_s + N_s (a_s r_s - w_0 e^{\frac{N_s}{N_s^{\text{th}}}} (r_s + \gamma_s)) \right). \quad (52)$$

If larger N_s leads to an overloaded system, the broadcast system usually increases the profit from the advertisement insertion. In other words, even when the cost increases due to overloading, the total profit can still be respectively increased. The complete algorithm herein is similar to the process of the first domain. The difference is the algorithm herein depends on the cost function. Algorithm 4 shows the details.

Algorithm 4. Optimal Permutation in the Third Domain

1: **Optimal permutation Π_{Ad}^***

$$\Pi_{\text{Ad}}^* = [\pi_{\text{Ad}}(1), \dots, \pi_{\text{Ad}}(S)],$$

where $m_{(\text{Ad})}(\tau_{\pi_{\text{Ad}}(s)}) = b_{\pi_{\text{Ad}}(s)}$, $b_{\pi_{\text{Ad}}(1)} > \dots > b_{\pi_{\text{Ad}}(S)}$

2: **Optimal permutation Π_r^***

$$\Pi_r^* = [\arg_{\pi(1)} \text{Profit}_{\pi(1)}^{\text{WSP}}, \dots, \arg_{\pi(S)} \text{Profit}_{\pi(S)}^{\text{WSP}}],$$

where $\text{Profit}^{\text{WSP(r)}} =$

$$\sum_{s=1}^S N_s (a_s r_s - w_0 e^{N_s/N_s^{\text{th}}} (r_s + \gamma_s)),$$

$$\forall s \in [1, S]$$

3: **Optimal permutation**

$$(\Pi_{\text{Ad}}^*, \Pi_r^*)$$

Similarly, the optimal proof follows Theorem 9. The marginal profits in the third domain are

$$m_{(\text{Ad})}(\tau_s) = b_s, \quad (53)$$

$$m_{(\text{r})}(r_s) = \left(a_s - \frac{w_0 e^{\frac{N_s}{N_s^{\text{th}}}}}{1-\varepsilon_s} \right) N_s. \quad (54)$$

5.4 Domain 4: Saturated $N_s > N_s^{\text{th}}$ and Nontolerable

$$\tau_s > \tau_s^{\text{th}}$$

To ensure the reception quality of each EU, the system should be adapted to the worst case because the rate allocation and advertisement scheduling are susceptible to such a

condition when $N_s > N_s^{\text{th}}$ and $\tau_s > \tau_s^{\text{th}}$ occur. However, in this case, we have $w_s(n_s) = w_0 e^{N_s/N_s^{\text{th}}}$, $n_s(\tau_s) = \frac{1}{\lambda_\rho(\tau_s+1)^\rho} \cdot N_s$, and $\tau_s = \tau_s^{\text{th}} + \Delta\tau_s^*$. In other words, the best profit can be obtained by computing a suitable $\Delta\tau_s^*$ even in the worst case. Like the second domain, the Lagrangian multipliers can be used in this domain. The following algorithm lists the steps.

Algorithm 5. Window Strategy for Worst Cases in the Fourth Domain

1: Characterize the decrement Δn_s via Eq. (16)

$$\begin{aligned} \Delta n_s &= n'_s(\tau_s) \Delta\tau_s \\ \text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s) &= b_s \Delta\tau_s - \\ &\left(1 - \frac{1}{\lambda_\rho(\Delta\tau_s+1)^\rho}\right) N_s r_s \left(a_s - w_0 e^{N_s/N_s^{\text{th}}} \frac{1}{1-\varepsilon_s}\right) \end{aligned}$$

2: Maximize $\text{Profit}_s^{\text{WSP}}(\Delta\tau_s, \Delta n_s)$ by using Lagrangian mutipliers

$$\begin{aligned} \Delta\tau_s^* &= \sqrt[\rho+1]{\frac{N_s r_s (a_s - w_0 e^{N_s/N_s^{\text{th}}} \frac{1}{1-\varepsilon_s}) \rho}{\lambda_\rho b_s}} - 1 \\ \Delta n_s^* &= n'_s(\tau_s) |_{\Delta\tau_s^*} \Delta\tau_s^* \end{aligned}$$

3: Obtain the optimal profit through the polymatroid structure

$$\begin{aligned} \text{Profit}^{\text{WSP}} &= \sum_{s=1}^S \left(b_s \tau_s^{\text{th}} + b_s \Delta\tau_s^* \right. \\ &\quad \left. + \sum_{n=1}^{N_s - \Delta n_s^*} (a_s r_s - w_0 e^{N_s/N_s^{\text{th}}} (r_s + \gamma_s)) \right) \end{aligned}$$

The profit maximization in the worst case is obtained by applying the optimal permutation mentioned in Algorithm 3. The optimal proof follows Theorem 12, and the marginal profits in the fourth domain are the same as those in the second domain.

5.5 Greedy Procedure in the General Case

We now consider the general case. Since the WSP is capable of controlling the access of all EUs, it means that the WSP manages the n_s and the τ_s by charging the EUs that are already serviced. Thus, it is possible to decide which domain the content s belongs to. This study uses the greedy algorithm (i.e., Algorithm 6) to solve the hybrid profit maximization mentioned in Eq. (18) by utilizing the polymatroidal structure.

Proposition 13. If $\text{Profit}^{\text{WSP}}(\mathbf{r}, \boldsymbol{\tau})$ is a polymatroid in \mathbb{R}_+ , then $\text{Profit}^{\text{WSP}}$ is greedy.

This proposition had been proved in [35].

Proposition 14. If $\text{Profit}^{\text{WSP(Ad)}}$ is a polymatroid, Algorithm 6 gives an optimum solution to Eq. (18).

The proof is referred to the proofs in [36].

Based on the above-mentioned two propositions and the polymatroidal structure, the greedy algorithm for optimal hybrid-domain allocation and scheduling can be derived.

6 NUMERICAL SIMULATION RESULTS

This section presents numerical examples by analyzing a simple wireless broadcasting network. This section also shows how to obtain optimal bandwidth allocation and

advertisement scheduling by using the models proposed in the above-mentioned sections.

Algorithm 6. Greedy (marginal) Algorithm in the General case

1: **Initialization:**

Define A_i as a set of video contents,
where all the elements fall in domain i and
 $i=1,2,3,4$, according to N_s and τ_s .
Partition $S = \{1, \dots, S\}$ into four subsets, i.e., A_i .

2: **Marginal Profit Computation:**

Switch A_i do

A_1 : Based on Eqs. (46) and (47), compute

$$\begin{aligned} \Pi_{\text{Ad}}^* &= \arg \max_{s \in A_1} (b_s) \\ \Pi_{\text{r}}^* &= \arg \max_{s \in A_1} (m_{(r)}(r_s)) \\ \Pi_1^* &= \Pi_{\text{Ad}}^* + \Pi_{\text{r}}^* \end{aligned}$$

A_2 : Based on Eqs. (50) and (51), compute

$$\Pi_2^* = \arg \max_{s \in A_2} \left(b_s + \sqrt{\frac{n_s(a_s - \frac{w_0}{1-\varepsilon_s})}{b_s r_s} + b_s n_s (a_s - \frac{w_0}{1-\varepsilon_s})} \right)$$

A_3 : Based on Eqs. (53) and (54), compute

$$\begin{aligned} \Pi_{\text{Ad}}^* &= \arg \max_{s \in A_3} (b_s) \\ \Pi_{\text{r}}^* &= \arg \max_{s \in A_3} (m_{(r)}(r_s)) \\ \Pi_3^* &= \Pi_{\text{Ad}}^* + \Pi_{\text{r}}^* \end{aligned}$$

A_4 : Based on Eqs. (50) and (51), compute

$$\begin{aligned} \Pi_4^* &= \arg \max_{s \in A_4} \left(b_s + \right. \\ &\quad \left. \sqrt{\frac{n_s(a_s - \frac{w_0 e^{N_s/N_s^{\text{th}}}}{1-\varepsilon_s})}{b_s r_s} + b_s n_s (a_s - \frac{w_0 e^{N_s/N_s^{\text{th}}}}{1-\varepsilon_s})} \right) \end{aligned}$$

3: Allocate Δr and $\Delta \tau$

Calculate $\Pi^* = \max\{\Pi_1^*, \Pi_2^*, \Pi_3^*, \Pi_4^*\}$

If Π^* is in the first or third domains

$$\Pi_{\text{Ad}}^* \leftarrow \tau_s + \Delta\tau, \Pi_{\text{r}}^* \leftarrow r_s + \Delta r$$

else if Π^* is in the second domain

$$\Pi_2^* \leftarrow \tau_s + \Delta\tau, \Pi_2^* \leftarrow r_s + \Delta r$$

else

$$\Pi_4^* \leftarrow \tau_s + \Delta\tau, \Pi_4^* \leftarrow r_s + \Delta r$$

6.1 Simulation Setup

Consider a wireless network with video broadcasting services. The video traffic and the connections of EUs are set up by a WSP. The devices of EUs are heterogeneous in terms of the content and channel conditions, i.e. various video contents and difference distances to the WSP. This simulation uses the H.264 extended SVC video encoder to generate layered video streams. Additionally, rateless error correction is used to for variable channel conditions. Assume that the system broadcasts three standard video sequences—"City," "soccer," and "harbor." Besides, the channel status of the users complies with the erasure-rate distribution [11]. Mobile EUs are requesting the service of different video contents, where the number of EUs and the number of video contents are exogenous variables.

The following part investigates the profit performance when the following different strategies are applied:

- (i) *Un_Weight* This is a widely used solution for resource allocation and scheduling. The broadcasting system employs a uniform model, where the limited bandwidth is uniformly distributed to S

contents. The duration of each inserted advertisement in each video content is also uniformly scheduled.

- (ii) *Ad_Weight* The allocation and scheduling follow the majority rule of bandwidth because the available bandwidth is the scarce in wireless environments. The third-generation (3G) and the long-term evolution (LTE) still benefit mainly from the traffic provision. Hence, the available bandwidth follows the common proportional allocation manner in these typical scenarios.
- (iii) *EU_Weight* The WSP earns profits mainly from the advertisement insertion. Free-WIFI access is broadly used in most wireless broadcasting systems. In this scenario, advertisement insertion becomes the major profit. The bandwidth allocation and advertisement scheduling depend on the number of EUs.
- (iv) *Weight_Ad_EU* Most WSPs provide the flexible free-*premium mode*, where parts of services are free of charge whereas the other services are not. In this mode, there is a tradeoff between the traffic provision and advertisement insertion. This tradeoff is modeled as a weight policy, $w_1 * Ad_Weight + w_2 * EU_Weight$.
- (v) *Proposed_Matroid*. The proposed profit maximization based on hybrid polymatroidal structure.

6.2 Profit Analysis in Four Domains

This subsection provides numerical examples to quantize the key properties of the proposed resource allocation strategy and the proposed advertisement-duration scheduling strategy.

6.2.1 Profit Comparison under Five Strategies

We firstly analyze the total profit $\text{Profit}^{\text{WSP}}$ of a WSP based on strategies (i)-(v) (see Section 6.1) in the aforementioned four domains. The entire results are respectively shown in Figs. 2a, 2d, 2g, and 2j. In the first domain, as shown in Fig. 2a, the working mode is typical for general broadcasting systems, where the number of EUs requesting services is not beyond the capacity of a system, and the advertisement duration is within a reasonable range. In this case, the profit $\Theta(\tau^*, \mathbf{r}^*)$ is directly maximized through two separate kinds of optimization — $\Theta_{(\text{Ad})}(\tau^*)$ and $\Theta_{(\text{r})}(\mathbf{r}^*)$. This is because the total profit is the summation result, $\text{Profit}^{\text{WSP}} = \text{Profit}^{\text{WSP}(\text{Ad})} + \text{Profit}^{\text{WSP}(\text{r})}$, and it does not conflict with $\text{Profit}^{\text{WSP}(\text{Ad})}$ and $\text{Profit}^{\text{WSP}(\text{r})}$. Similar result is found in the third domain (see Fig. 2g). When the broadcasting system becomes overloaded, it can adjust the operation mode through the control of n_s . Then, the profit is maximized by the boundary surface achieving. In these two cases, the difficulty is to rapidly find the optimal vector \mathbf{r}^* and τ^* in variable circumstances. Through polymatroid modeling, the optimal results can be explicitly obtained in a greedy manner by finding optimal permutation, illustrated in Algorithms 1 and 4. When advertisement duration τ_s gradually increases, the WSP begins to lose their users, which implies that n_s decreases. Interestingly, inflection points appear in both Figs. 2d and 2j, which respectively reflect the results of the

second and fourth domains. In these cases, the problem becomes how to determine $\Delta\tau_s^*$ when the maximum profit occurs. Rather than obtaining the maximum profit in the threshold τ_s^{th} or N_s^{th} , the maximum profit is achieved when $\tau_s > \tau_s^{\text{th}}$ and $n_s > N_s^{\text{th}}$ hold. Thus, this correspondingly indicates it is a hybrid-domain decision between τ_s and r_s . The optimization solution is found at the extreme point of the polymatroidal structure after permutation is performed. Namely, the maximum profit solution is found by the boundary surface achieving. If the problem belongs to the second domain, the optimal \mathbf{r}^* and τ^* can be calculated by using Algorithms 2 and 3. As for those in the fourth domain, Algorithm 5 becomes applicable during maximization. The profit under the policy (v) is larger than those under the other four policies because both $\text{Profit}^{\text{WSP}(\text{Ad})}$ and $\text{Profit}^{\text{WSP}(\text{r})}$ in policy (v) are better than those in the other policies. The following two subsections show how the profit changes when different policies are applied.

6.2.2 Profit Performance Comparison under Policy \mathcal{R}_r

Figs. 2b, 2e, 2h, and 2k illustrate the $\text{Profit}^{\text{WSP}(\text{r})}$ under five different policies. The profit performance of the WSP under the proposed scheme (v) outperforms those under the other four schemes. These results imply four conclusions. 1) In multicontent broadcasting, rate allocation should consider not only the contents but also EUs. This is because the results under policy (v) show the best profit in most cases. 2) In multiuser systems, profits are not improved with more consideration of EUs' requirements. Although, user-centric design typically dominates service providers, however, the results under policy (iii) do not present favorable performance compared with those under the other policies. This shows that WSP cannot generate more benefits simply from EUs' satisfaction because it is difficult to simultaneously satisfy the diverse demands from all EUs. 3) In profit-driven systems, advertisement insertion is the most profitable mode because the policy (ii) is better than policies (i), (iii) and (iv). These results also demonstrate how current mobile streaming services should be operated—Generate revenue primarily by delivering relevant and cost-effective online advertising. 4) Suitable optimal structures help improve the profit performances of the WSP. The proposed policy (v) shows better performance in two aspects. First, the proposed scheme keeps the best profit in a wide range of available bandwidth. Second, the proposed scheme can make the WSP obtain the profit quickly because the largest profits are achieved in a lower available broadcasting rate.

6.2.3 Profit Performance Comparison under Scheduling Policy \mathcal{R}_τ

Figs. 2c, 2f, 2i, and 2l demonstrate the $\text{Profit}^{\text{WSP}(\text{Ad})}$ under the five policies. The profit performance of the WSP under the proposed scheme (v) still exceeds those under the other four schemes. These results reflect two conclusions. 1) To control the advertisement duration adaptively does not always improve the system profit because the results under policy (ii) show worse profits in most cases. The phenomenon reflects that profits from advertisements becomes complex in the wireless scenario because of the joint pricing

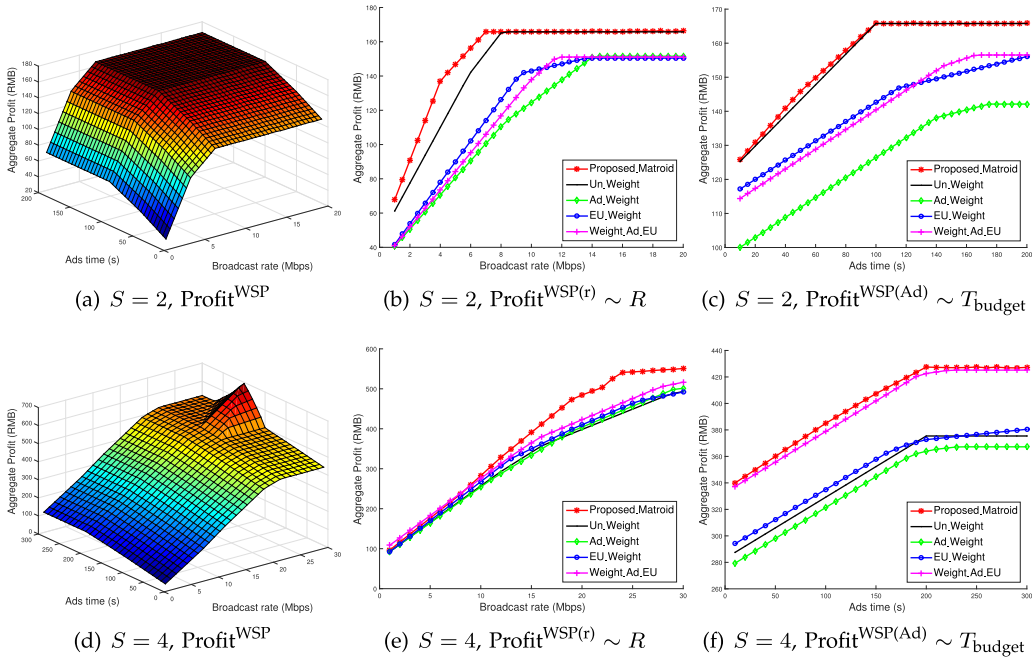


Fig. 3. Performance comparison for variable contents for wireless video broadcasting.

between advertisements and traffic provision. 2) Insertion of long-duration advertisements does not improve the system profit, as shown in Figs. 2f and 2l. In these two cases, $\tau_s > \tau_s^{\text{th}}$ leads to the rapid decrease of n_s such that the profit Profit^{WSP(Ad)} also reduces quickly. Consequently, there is an obvious optimum point between n_s and τ_s . 3) Proposed polymatroid-based method exhibits flexible performance because the results under policy (v) show the best profit in every domain even when $\tau_s > \tau_s^{\text{th}}$.

For the performance when the number of video content s increases (Fig. 3), the main trends and conclusions are similar. Fig. 3 demonstrates the total profit Profit^{WSP} of a WSP, the respective profits Profit^{WSP(r)} and Profit^{WSP(Ad)} under the $S = 2$ (Figs. 3a, 3b, 3c) and $S = 4$ (Figs. 3d, 3e, 3f). It is possible to observe that, the proposed policy (v) keeps the best profit performance when the number of contents increases and when the number of EUs varies.

6.3 Complexity Analysis

This subsection examines the complexity of the proposed approach. It investigates whether it is efficient or not when polymatroid-based approach is used in the wireless video broadcasting system. Generally speaking, one major advantage of the polymatroid-based approach depends on its property. That is, each maximal independent set is also the maximum independent set. Consequently, the optimal rate allocation and the advertisement scheduling in each domain can be explicitly obtained in a greedy manner. The next part analyzes the complexity of the propose approach in four domains.

In the first domain, for subproblem $\Theta_{(\text{Ad})}(\tau)$, the computational complexity is $O(S \log S)$ because the computation mainly lies in the marginal profit permutation due to insertion of S advertisements. For the subproblem $\Theta_{(r)}(\mathbf{r})$, the computational complexity is also $O(S \log S)$ because the permutation process for S contents is equivalent to marginal

profit either for advertisement insertion or rate allocation. Thus, the overall computational complexity in the first domain is $O(S \log S)$.

In the second domain, according to Algorithm 2, the rate is allocated into many windows with the size of $\Delta\tau_s$. Let $K = \lfloor \frac{R}{\min_s \Delta\tau_s} \rfloor$ be the upper bound of the number of windows. In each window, the system ranks the broadcasting rate in order according to the marginal profit. Consequently, the major computation focuses on the optimal permutation of rate allocation. The overall complexity of the proposed approach is no more than $O(KS \log S)$.

Like the results in the previous two domains, the complexity in the third and the fourth domain is respectively $O(S \log S)$ and $O(KS \log S)$.

7 CONCLUSION AND FUTURE WORK

In order for the WSP to gain the maximum profit in advertisement insertion, this study analyzes the interplay between traffic provision and advertisement insertion. The maximum achievable profit depends on joint optimization of the optimal rate vector and the optimal advertisement-duration vector. Optimization of these two vectors is usually computationally intensive. To resolve this problem, this study first formulates the overall profit as the solution to four suboptimization problems via horizontal and vertical decomposition. Then, by means of a theoretic polymatroidal framework, this study shows that the optimal profit points can be obtained by using a hierarchical greedy algorithm. Simulation results show that the proposed solution can improve the system profit effectively compared with the other four baselines. In the future work, we plan to explore how to take into account user preferences in our optimization formulation. For example, user profiling based on personalized advertisement insertion could lead to more efficient models and to yield more profits.

APPENDIX A PROOF OF THEOREM 1

Under a given policy \mathcal{R}_r , $\mathbf{r} = [r_1, \dots, r_S]$ is then fixed. The resulting $\mathbf{P}^{\text{WSP}(\mathbf{r})}$ is analyzed as follows.

1) It is normalized: Clearly, $\mathbf{P}^{\text{WSP}(\mathbf{r})}(\emptyset) = 0$.

2) It is nondecreasing:

Let T and T' be two finite sets of elements in E .

Besides, $T' \subseteq T \subseteq E = \{1, 2, \dots, S\}$. Then,

$$\mathbf{P}^{\text{WSP}(\mathbf{r})}(T') = \sum_{s \in |T'|} P_s^{\text{WSP}(\mathbf{r})}.$$

$$\begin{aligned} \mathbf{P}^{\text{WSP}(\mathbf{r})}(T) &= \sum_{s \in |T|} P_s^{\text{WSP}(\mathbf{r})} = \sum_{s \in |T'|} P_s^{\text{WSP}(\mathbf{r})} + \sum_{s \in |T| - |T'|} P_s^{\text{WSP}(\mathbf{r})} \\ &\geq \mathbf{P}^{\text{WSP}(\mathbf{r})}(T'). \end{aligned}$$

3) It is submodular:

Let T' and T'' be any two finite sets of elements in E .

Therefore, there are $T' \subseteq E = \{1, 2, \dots, S\}$ and $T'' \subseteq E = \{1, 2, \dots, S\}$. Then,

$$\begin{aligned} &\mathbf{P}^{\text{WSP}(\mathbf{r})}(T') + \mathbf{P}^{\text{WSP}(\mathbf{r})}(T'') \\ &= \sum_{s \in |T'|} P_s^{\text{WSP}(\mathbf{r})} + \sum_{s \in |T''|} P_s^{\text{WSP}(\mathbf{r})} \\ &= \sum_{s \in T' \cap T''} P_s^{\text{WSP}(\mathbf{r})} + \sum_{s \in T' - T''} P_s^{\text{WSP}(\mathbf{r})} + \sum_{s \in T'' - T'} P_s^{\text{WSP}(\mathbf{r})} \\ &= \sum_{s \in T' \cap T''} P_s^{\text{WSP}(\mathbf{r})} + \sum_{s \in T' \cup T''} P_s^{\text{WSP}(\mathbf{r})} \\ &= \mathbf{P}^{\text{WSP}(\mathbf{r})}(T' \cap T'') + \mathbf{P}^{\text{WSP}(\mathbf{r})}(T' \cup T''). \end{aligned}$$

Thus, $\mathbf{P}^{\text{WSP}(\mathbf{r})}$ satisfies the submodular property.

This completes the proof. \square

APPENDIX B PROOF OF THEOREM 4

For any policy F^k , where $F^k \in \mathcal{F}$, denote \mathbf{r}^k as the corresponding allocated rate vector. The resulting profit can be written as

$$\begin{aligned} \mathcal{P}^{\text{WSP}(\mathbf{r}^k)}(\mathcal{R}) &= \left\{ \mathbf{P}^{\text{WSP}(\mathbf{r})} : \mathbf{P}^{\text{WSP}(\mathbf{r})}(\mathbf{r}^k) \right. \\ &\quad \left. \preceq \frac{1}{\lambda_\rho (\tau_s + 1)^\rho} N_s (a_s r_s^k - w_0 (r_s^k + \gamma_s^k)), \forall s \in S \right\}. \end{aligned}$$

The boundary surface of $\mathcal{P}^{\text{WSP}(\mathbf{r})}(\mathcal{R})$ is

$$\mathcal{P}^{\text{WSP}(\mathbf{r})}(\mathcal{R}) \subset \bigcup_{\mathcal{R} \in F^k} \mathcal{P}^{\text{WSP}(\mathbf{r}^k)}(\mathcal{R}).$$

Combining the above results yields

$$\bigcup_{\mathcal{R} \in F^k} \mathcal{P}^{\text{WSP}(\mathbf{r})}(\mathcal{R}) \subset \bigcup_{\mathcal{R} \in F} \mathcal{P}^{\text{WSP}(\mathbf{r})}(\mathcal{R}) \subset \bigcup_{\mathcal{R} \in F^k} \mathcal{P}^{\text{WSP}(\mathbf{r}^k)}(\mathcal{R}).$$

When $k \rightarrow \infty$, $\bigcup_{\mathcal{R} \in F} \mathcal{P}^{\text{WSP}(\mathbf{r})}(\mathcal{R}) \rightarrow \bigcup_{\mathcal{R} \in F} \mathcal{P}^{\text{WSP}(\mathbf{r}^k)}(\mathcal{R})$.

Consequently, $\mathcal{P}^{\text{WSP}(\mathbf{r})}(\bar{\mathbf{r}}) \rightarrow \bigcup_{\mathcal{R} \in F} \mathcal{P}^{\text{WSP}(\mathbf{r})}(\mathcal{R})$.

This completes the proof. \square

APPENDIX C PROOF OF THEOREM 8

The original profit maximization problem can be rewritten as

$$\max_{(\boldsymbol{\tau}, \mathbf{r})} \mathbf{P}^{\text{WSP}} \text{ s.t. } (\boldsymbol{\tau}, \mathbf{r}) \in (\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}}). \quad (55)$$

Using Lagrangian formulation yields

$$\max_{(\boldsymbol{\tau}, \mathbf{r})} \mathbf{P}^{\text{WSP}} - \lambda_1 \boldsymbol{\tau} - \lambda_2 \mathbf{r}.$$

since

$$\begin{aligned} \mathcal{P}^{\text{WSP}}(\bar{\boldsymbol{\tau}}, \bar{\mathbf{r}}) &= \bigcup_{\boldsymbol{\tau}: \sum_s \tau_s \leq \bar{\boldsymbol{\tau}} \cdot \mathbf{1}; \mathbf{r}: \sum_s r_s \leq \bar{\mathbf{r}} \cdot \mathbf{1}} \mathbf{P}^{\text{WSP}}(\boldsymbol{\tau}, \mathbf{r}) \\ &= \bigcup_{\boldsymbol{\tau}: \sum_s \tau_s = \bar{\boldsymbol{\tau}} \cdot \mathbf{1}; \mathbf{r}: \sum_s r_s = \bar{\mathbf{r}} \cdot \mathbf{1}} \mathbf{P}^{\text{WSP}}(\boldsymbol{\tau}, \mathbf{r}). \end{aligned}$$

Denote $\pi(k)$ as the permutation of the components within a content vector in a decreasing order, where $\tau_{k-1} \preceq \tau_k$ and $r_{k-1} \preceq r_k$. Therefore,

$$\begin{aligned} \mathbf{P}_{\pi(k)}^{\text{WSP}} &= u_{\pi(k)}^{\text{WSP(Ad)}}(\boldsymbol{\tau}_{\pi(k)}) \\ &\quad + \sum_{n=1}^{N_{\pi(k)}(\boldsymbol{\tau})} (u_{\pi(k),n}^{\text{WSP}(\mathbf{r})}(r_{\pi(k)}) - e^{-\frac{N_{\pi(k)}(\boldsymbol{\tau})}{N_0}} (r_{\pi(k)} + \gamma_{\pi(k)})), \quad \forall k \\ &\in \{1, \dots, S\}. \end{aligned}$$

Then, Eq. (55) can be rewritten as

$$\begin{aligned} &\max_{(\boldsymbol{\tau}, \mathbf{r})} \sum_{k=1}^S (u_{\pi(k)}^{\text{WSP(Ad)}}(\boldsymbol{\tau}_{\pi(k)}) \\ &\quad + \sum_{n=1}^{N_{\pi(k)}(\boldsymbol{\tau})} (u_{\pi(k),n}^{\text{WSP}(\mathbf{r})}(r_{\pi(k)}) - e^{-\frac{N_{\pi(k)}(\boldsymbol{\tau})}{N_0}} (r_{\pi(k)} + \gamma_{\pi(k)}))) - \lambda_1 \bar{\boldsymbol{\tau}} \\ &\quad - \lambda_2 \bar{\mathbf{r}}. \end{aligned}$$

This completes the proof. \square

APPENDIX D PROOF OF THEOREM 9

For the objective function in Eq. (18), there is

$$\begin{aligned} \Theta(\boldsymbol{\tau}^*, \mathbf{r}^*) &\stackrel{(a)}{=} \max \sum_{s=1}^S \left(b_s \tau_s^* + \sum_{n=1}^{n_s} a_s r_s^* - w_s(n_s) \cdot (r_s^* + \gamma_s^*) \right) \\ &\stackrel{(b)}{=} \max \sum_{s=1}^S b_s \tau_s^* \\ &\quad + \max \sum_{s=1}^S \sum_{n=1}^{n_s} (a_s r_s^* - w_s(n_s) \cdot (r_s^* + \gamma_s^*)) \\ &\stackrel{(c)}{=} \Theta_{(\text{Ad})}(\boldsymbol{\tau}^*) + \Theta_{(\mathbf{r})}(\mathbf{r}^*). \end{aligned}$$

- (a) According to the definition of $\Theta(\mathbf{r}, \tau)$, $\Theta(\mathbf{r}, \tau) = \max_{\mathbf{r}, \tau} [\text{Profit}^{\text{WSP}}]$.
- (b) In the first domain, as the variation of τ and \mathbf{r} do not affect n_s , subsequently $\gamma_s^* = r_s^* \left(\frac{1+\delta}{1-\epsilon_s n_s} - 1 \right)$ according to Eq. (23).
- (c) Based on the definitions of $\Theta_{(r)}(\mathbf{r})$ and $\Theta_{(Ad)}(\tau)$, we obtain $\Theta_{(r)}(\mathbf{r}) = \max_{\mathbf{r}} [\text{Profit}^{\text{WSP}(r)}]$ and $\Theta_{(Ad)}(\tau) = \max_{\tau} [\text{Profit}^{\text{WSP}(Ad)}]$.
- This completes the proof. \square

APPENDIX E PROOF OF LEMMA 10

Let $\tau_s = \tau_s^{\text{th}} + \Delta\tau_s$. Combining Eq. (16) with Eq. (18) yields

$$\begin{aligned} \text{Profit}^{\text{WSP}}(\mathbf{r}, \tau) &= \sum_{s=1}^S \left(b_s \tau_s + \sum_{n=1}^{n_s(\tau)} (a_s r_s - w_s(r_s + \gamma_s)) \right) \\ &= \sum_{s=1}^S \left(b_s \tau_s^{\text{th}} + b_s \Delta\tau_s + \sum_{n=1}^{N_s - \Delta n_s} (a_s r_s - w_s(r_s + \gamma_s)) \right) \\ &= \sum_{s=1}^S \left(b_s \tau_s^{\text{th}} + \sum_{n=1}^{N_s} (a_s r_s - w_s(r_s + \gamma_s)) \right) \\ &\quad + \sum_{s=1}^S \left(b_s \Delta\tau_s - \sum_{n=1}^{\Delta n_s} (a_s r_s - w_s(r_s + \gamma_s)) \right) \\ &= \text{Profit}^{\text{WSP}}(\tau^{\text{th}}, \mathbf{N}) + \text{Profit}^{\text{WSP}}(\Delta\tau, \Delta\mathbf{n}). \end{aligned}$$

This completes the proof. \square

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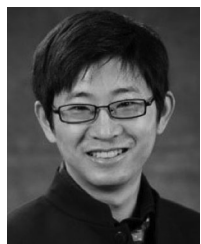
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